ON NUCLEAR BINDING ENERGIES

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In a previous communication. (Dutta and others, 1963) the relationship for the binding energies of the most strongly bound nuclei of different mass numbers have been obtained in the form :

$$E = B(A)+F(Z)+F(I), \qquad \text{... (a)}$$

a combination of base function and two periodic functions in nearly opposite phase.

It had been indicated that some modification would be necessary to make them correspond to the nuclei with optimum energy and optimum neutron number. Modifications have also been found necessary to satisfy the requirements of the weakly bound nuclei and excited nuclei, where one of these periodic functions, $F(Z)$, plays an important role. The modified equations, given later on, obtain the positions of the minima and the maxima as well as the associated amplitudes, slightly varying from the previously obtained values. The maxima, and minima positions in mass numbers of the periodic function $F(Z)$ and the amplitude a_z, at the corresponding positions, are tabulated below :

	min	max	min	max	min	max	min	max	min	max
$A(F_z)$	16	27	40	61	90	117	140	177	208	237
a_z	16.7	10.9	10.9	10.9	16.7	10.9	12.5	10.9	16.7	10.9

The binding energies of the set of nuclei of a particular odd mass number as well as those for the even-even nuclei of an even mass number, are known to be determined by a relation of the approximate form :

$$E = E_0+\beta(N-N_0)^2 \qquad \text{... (b)}$$

where N_0 is the optimum neutron number for the given mass number and β is the neutron proton exchange energy. It is also known that a simple quadratic relation of this type, with constant E_0, β and N_0 values, does not work satisfactorily for all the nuclei of a given mass number, when the isobaric nuclei are large in number. We have kept the quadratic form and have tried to obtain relations for the changes in β and N_0 values with N.

In accordance with the quadratic form of relationship, the β-values for odd mass nuclei are determined by the mean deviation of the energies of the nearest isobaric neighbours on the two sides, from the energy of the most strongly bound one, obtainable from available tables (Konig *et al.* 1962). Thus, when x, $1-x$ and $1+x$ are the fractional and nonintregral numbers to measure the changes in neutron numbers of the most strongly bound one and its neighbours from the optimum neutron number, the mean energy deviation from that for the most strongly bound nuclei would be given by

$$\tfrac{1}{2}\beta\{(1+x)^2-x^2+(1-x)^2-x^2\} = \beta \tag{c}$$

The N_0 values corresponding to it and named as N_0^* are determined from the relationship,

$$N_0^* = \tfrac{1}{2}(N_1+N_2)-\frac{1}{2\beta}(E_2-E_1) \tag{d}$$

which follows from the relationship,

$$E_2-E_1 = \beta\{(N_2-N_0)^2-(N_0-N_1)^2\}$$

a consequence of relation (b) with $N_2 = N_1+1$.

It has been obtained that the β values and the N_0^* values, calculated from the nearest neighbours of the odd-mass nuclei are composed of two parts : a function $\beta(A)$ or $N_0^*(A)$ dependent on mass numbers only, superposed by contributions of a periodic function, $\beta(S)$ or $N_0^*(S)$. $\beta(S)$ have been subdivided into three components $\beta(S_1)$, $\beta(S_2)$ and $\beta(S_3)$. The periodic function $F(Z)$ with modulated amplitude gives the $\beta(S_1)$ values There is a strong enhancement of the $\beta(S)$ values at the minima of the function $F(Z)$ which is represented as $\beta(S_2)$. A combinational effect of the maxima of the $F(Z)$ function and minima of the $F(I)$ function, give further minor and irregular decrease in β-values near $F(Z)$ maxima positions, represented by $\beta(S_3)$. The effect of $F(I)$ minima may be better described by specifying the nuclei, that comes back to the periodic curve maxima.

The β-values calculated from next to nearest neighbour for odd masses, as also from the even-even isobaric nuclei, generally, agree. It also shows that $\beta(S)$-part of the β –values decrease with larger $\Delta N = |N_0-N|$ values. It is expressed by the factor $\sigma(\Delta N)$, in the following. It implies that the effect of the superposed periodic structure becomes weaker, as we go away from the optimum-neutron-number condition of the nuclei.

It has also been noted that N_0 values change to larger magnitudes as we increase ΔN values, implying a larger percentage of optimum neutron number, corresponding to weakly bound nuclei.

For the odd-odd nuclei, we can take the N_0^* value as the mean of the values of the previous and following odd-mass nuclei. It enables one to calculate β and E'_0 values from the two equations obtained from relation (b). The β values, so obtained. are in general agreement with the values expected from the previous and the following mass number's β values. These β and N_0^* values enable one to calculate the E_0 value for the even-even nucleus. It gives us the correction $C_{00} = E_0 - E'_0$, for the odd-odd nuclei of any particular even-mass number.

It may be observed that the enhancement and decrease of β values at the minima and maxima of the $F(Z)$ curve, is alike in nature to general excitation characteristics associated with potential energy levels and indicates the possibility of correlating nuclear excitation with the β-values. This has been taken up in the next communication.

The complete set of relations and the tabulated experimental and calculated values for all the isobars of some mass numbers are given below, as prototypes. Small adjustments of the relations are expected to give a closer agreement.

Relations

$$E = E_0 + \beta(N - N_0)^2 \tag{0}$$

$$= B(A) + F_z + F_I + \beta(N - N_0)^2$$

$$= B(A) + a_z \cos \pi f(Z) - a_I \cos \pi \{f(Z) + \varphi\}$$

$$+ \{\beta(A) + \sigma(\Delta N).\Sigma\beta(S_i)\}.\{N - N_0^* + \eta(\Delta N)\}^2$$

where,

$$B(A) = -9.828A + 8.877 \times 10^{-3} A^2 + C_{ij} \text{ mev.} \tag{1}$$

$$C(ee) = 32.2;\ C\binom{eo}{oe} = 33.0;\ C(oo) = 34 + 80A^{-1}$$

$$N_0^* = .6302A - 0.1287A \cdot \exp(-7.95 \times 10^{-3} \cdot A) - .00155A \times \tag{2}$$

$$\cos\pi\{0.794 \sinh .0372(A - 104)\}\{1 - \tanh .6(A - 45)\}$$

$$\times\{1 - \tanh .6(A - 145)\}$$

$$\eta(\Delta N) = 0.1\,|N_0^* - N| - 0.1$$

It gives the increase in percentage of neutron number from about 50 at lower mass numbers to 63 for higher mass numbers with a superposed periodic variation in the range of mass numbers 40 to 150.

$$f(Z) = -051 + .0339A - 2 \exp - 1.18 \times 10^{-3}.A^2 + 0.3. \exp - 3.43$$

$$\times 10^{-3}.(A - 140)^2.$$

$$a_z = 10.9 - 2.9\,[\sin\pi\ 0.5f(Z) - \Sigma_i \exp - \alpha_i(A - A_i(F_z.\min)^2] \tag{3}$$

$$a_I = 10 + 2.9 \sin\pi\,(.008A) + 2.3 \exp - 2 \times 10^{-3}\,(A - 200)^2$$

$$\phi = -0.11 + .135 \cos\pi\,\{a_i \sinh b_i(A - A_{oi})\}$$

[$\alpha_i = \dfrac{200}{T^2}$; T = period in mass number of F_z, at the zone concerned. a_i, b_i are associated constants with A_{oi} for one period about A_{oi} only. Associated constants (A_{oi}; b_i; a_i) are (70;3.42×10^{-2}; 0.106); (186; 7.317×10^{-2}; 0.226) and (256; 8.625×10^{-2}; 0.0636)]

(4) $$\beta(A) = 67.A^{-1} \exp b(A-52) \qquad b = \begin{Bmatrix} -.01 \text{ for } A<52 \\ +.0038 \text{ for } A>52 \end{Bmatrix}$$

$$\sigma(\Delta N) = 0.6+0.6 \tanh (1.6-8\Delta N)$$

$$\beta(S_1) = \{0.95-.6 \tanh .06(A-30)-.25 \tanh .12(A-145)\} \cos \pi f(Z)$$

$$\beta(S_2) = .08\, a_z \exp -\gamma_i\{A-A_i(F_z \min)\}^2$$

$$\beta(S_3) = -\{.45-.15 \tanh (A-140)\} \exp -\gamma_i\{A'-A_i(F_z \max)\}^2$$

[$\gamma_i = 80/T^2 A'$ refers to mass numbers 27, 31; 59; 113-115, which are affected by F_z max Other nuclei in the regions obtain balanced effect due to F_z max and F_I min.]

TABLE I

Element	A	-E (exp) mev.	-E (Cal) mev.	Element	A	-E (exp) mev.	-E (Cal) mev.	Element	A	-E (exp) mev.	-E (Cal) mev.
C	15	106.5	108.3	Se	74	642.9	642.7	Er	170	1379.0	1379.8
N	15	115.5	116.7	Br	74	636.1	635.1	Tm	170	1377.8	1377.6
O	15	111.9	114.4	Kr	74	631.2	630.8	Yb	170	1378.0	1378.7
								Lu	170	1373.7	1373.6
C	16	110.8	112.5	Br	85	737.4	736.0				
N	16	118.0	118.3	Kr	85	739.5	738.6				
O	16	127.6	128.1	Rb	85	739.3	738.0	Tl	210	1640.9	1640.3
F	16	111.2	111.9	Sr	85	737.5	737.6	Pb	210	1645.6	1645.8
								Bi	210	1644.8	1644.5
S	35	298.8	297.6	Y	85	733.7	734.2	Po	210	1645.2	1644.0
Cl	35	298.2	297.3	Mo	99	852.0	854.1	At	210	1640.6	1639.6
Ar	35	291.4	292.4	Tc	99	852.6	854.4	Rn	210	1637.3	1637.1
Sc	50	432.3	431.8	Ru	99	852.1	853.5	Pa	235	1783.2	1784.2
Ti	50	437.8	437.8	Rh	99	849.2	850.3	U	235	1783.8	1784.2
V	50	434.8	434.6	Pd	99	844.6	845.3	Np	235	1782.9	1783.4
Cr	50	435.0	434.9	Te	130	1095.5	1096.3	Pu	235	1781.0	1780.8
Mn	50	426.9	425.3	I	130	1094.7	1094.8				
								Am	245	1841.4	1841.3
Ga	74	640.8	640.5	Xe	130	1096.9	1097.0	Cm	245	1841.5	1840.9
Ge	74	645.7	645.2	Cs	130	1093.1	1092.8	Bk	245	1839.9	1839.0
As	74	642.3	642.3	Ba	130	1092.8	1091.6	Cf	245	1837.6	1836.9

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